1989 - AB1

1. Let f be the function given by $f(x) = x^3 - 7x + 6$.

   (a) Find the zeros of f.

   (b) Write an equation of the line tangent to the graph of f at $x = -1$.

   (c) Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 3]$.

1989 - AB2

2. Let R be the region in the first quadrant enclosed by the graph of $y = \sqrt{6x + 4}$, the line $y = 2x$, and the y-axis.

   (a) Find the area of R.

   (b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the x-axis.

   (c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

1989 - AB3

3. A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t) = 4\cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0) = 1$ and its position is $x(0) = 0$.

   (a) Write an equation for the velocity $v(t)$ of the particle.

   (b) Write an equation for the position $x(t)$ of the particle.

   (c) For what values of t, $0 \leq t \leq \pi$, is the particle at rest?

1989 – AB 4

4. Let f be the function given by $f(x) = \dfrac{x}{\sqrt{x^2 - 4}}$.

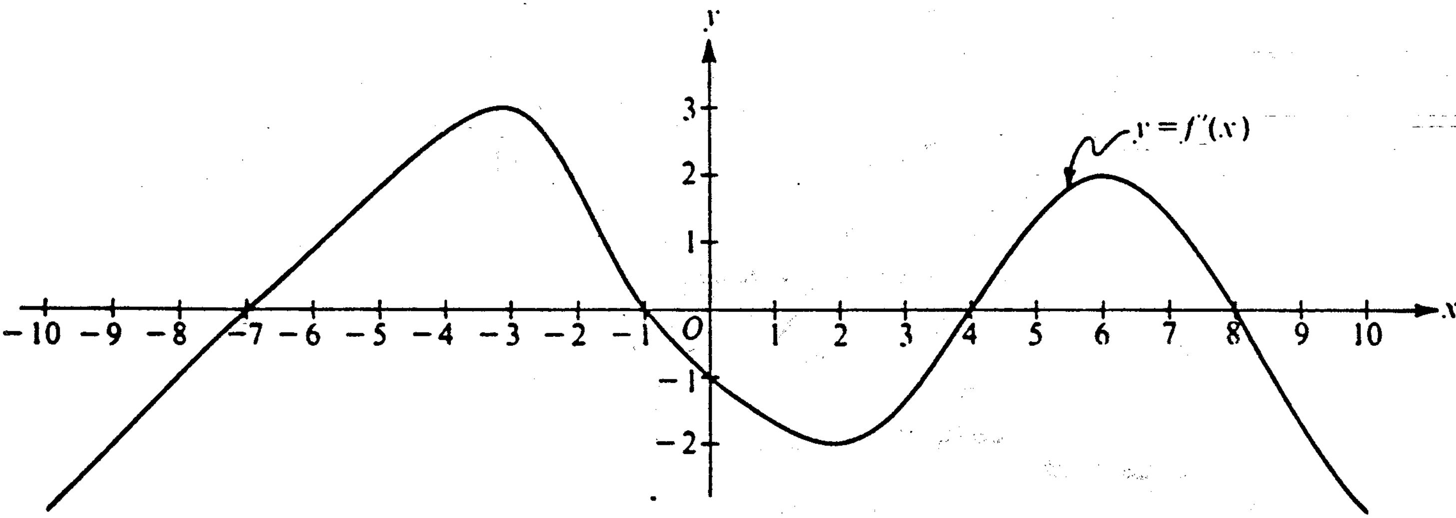
(a) Find the domain of f.

(b) Write an equation for each vertical asymptote to the graph of f.

(c) Write an equation for each horizontal asymptote to the graph of f.

(d) Find $f'(x)$.

1989 – AB 5



<u>Note:</u> This is the graph of the <u>derivative</u> of f, <u>not</u> the graph of f.

5. The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \leqq x \leqq 10$.

(a) For what values of x does the graph of f have a horizontal tangent?

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer.

(c) For what values of x is the graph of f concave downward?

1989 – AB 6

6. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

   (a) Write an equation for y, the amount of oil remaining in the well at any time t.

   (b) At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

   (c) In order not to lose money, at what time t should oil no longer be pumped from the well?

1989 – BC 1

1. Let f be a function such that $f''(x) = 6x + 8$.

   (a) Find $f(x)$ if the graph of f is tangent to the line $3x - y = 2$ at the point $(0, -2)$.

   (b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

1989 – BC 2

2. Let R be the region enclosed by the graph of $y = \dfrac{x^2}{x^2 + 1}$, the line $x = 1$, and the x-axis.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the <u>y-axis</u>.

1989 – BC 3

3. Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

   (a) Find the absolute maximum and minimum values of $f(x)$.

   (b) Find the intervals on which f is increasing.

   (c) Find the x-coordinate of each point of inflection of the graph of f.

1989 – BC 4

4. Consider the curve given by the parametric equations

$$x = 2t^3 - 3t^2 \quad \text{and} \quad y = t^3 - 12t.$$

(a) In terms of t, find $\frac{dy}{dx}$.
(b) Write an equation for the line tangent to the curve at the point where $t = -1$.
(c) Find the x- and y-coordinates for each critical point on the curve and identify each point as having a vertical or horizontal tangent.

1989 – BC 5

5. At any time $t \geqq 0$, the velocity of a particle traveling along the x-axis is given by the differential equation $\frac{dx}{dt} - 10x = 60e^{4t}$.

(a) Find the general solution $x(t)$ for the position of the particle.
(b) If the position of the particle at time $t = 0$ is $x = -8$, find the particular solution $x(t)$ for the position of the particle.
(c) Use the particular solution from part(b) to find the time at which the particle is at rest.

1989 – BC 6

6. Let f be a function that is everywhere differentiable and that has the following properties.

(i) $f(x+h) = \dfrac{f(x) + f(h)}{f(-x) + f(-h)}$ for all real numbers h and x.

(ii) $f(x) > 0$ for all real numbers x.

(iii) $f'(0) = -1$.

(a) Find the value of $f(0)$.
(b) Show that $f(-x) = \dfrac{1}{f(x)}$ for all real numbers x.
(c) Using part (b), show that $f(x+h) = f(x)f(h)$ for all real numbers h and x.
(d) Use the definition of the derivative to find $f'(x)$ in terms of $f(x)$.